Please check the examination details below before entering your candidate information

| Candidate surname | Other names |
|---|---|
| | |

Centre Number | Candidate Number

■ : explanation
∵ is 'because'
∴ is 'therefore'

# Pearson Edexcel International Advanced Level

**Monday 9 October 2023**

Afternoon (Time: 1 hour 30 minutes)

**Paper reference** **WMA11/01**

# Mathematics

**International Advanced Subsidiary/Advanced Level**
**Pure Mathematics P1**

**You must have:**
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

**Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.**

## Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

## Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 11 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

## Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

*Turn over* ▶

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Z:1/1/1/

P 7 4 3 1 6 A 0 1 3 2

Pearson

1. Given that

$$y = 5x^3 + \frac{3}{x^2} - 7x \qquad x > 0$$

find, in simplest form,

(a) $\frac{dy}{dx}$ **(3)**

(b) $\frac{d^2y}{dx^2}$ **(2)**

$y \xrightarrow{\text{differentiate}} dy/dx \xrightarrow{\text{differentiate}} d^2y/dx^2$

a) ① Write in easier form for differentiation

$y = 5x^3 + \frac{3}{x^2} - 7x = 5x^3 + 3x^{-2} - 7x^1$

* indicies rule : $\frac{a}{x^b} = ax^{-b}$

② differentiate

$dy/dx = 3(5x^{3-1}) + (-2)(3x^{-2-1}) + 1(-7x^{1-1}) = 15x^2 - 6x^{-3} - 7$

$\therefore dy/dx = 15x^2 - 6x^{-3} - 7$

b) differentiate $dy/dx$ : $dy/dx = 15x^2 - 6x^{-3} - 7x^0$ $\because x^0 = 1$

$d^2y/dx^2 = 2(15x^{2-1}) + (-3)(-6x^{-3-1}) + 0(-7x^{0-1}) = 30x + 18x^{-4}$

$\therefore d^2y/dx^2 = 30x + 18x^{-4}$

**Question 1 continued**

**(Total for Question 1 is 5 marks)**

**2.** Given that

$$a = \frac{1}{64}x^2 \qquad b = \frac{16}{\sqrt{x}}$$

express each of the following in the form $kx^n$ where $k$ and $n$ are simplified constants.

(a) $a^{\frac{1}{2}}$

**(1)**

(b) $\dfrac{16}{b^3}$

**(1)**

(c) $\left(\dfrac{ab}{2}\right)^{-\frac{4}{3}}$

**(2)**

a) $a^{\frac{1}{2}} = \left(\frac{1}{64}x^2\right)^{\frac{1}{2}} = \sqrt[2]{\left(\frac{1}{64}x^2\right)^1} = \sqrt{\frac{1}{64}x^2} = \sqrt{\frac{1}{64}} \times \sqrt{x^2}$

indicies rule: $\sqrt[c]{a^b} = a^{\frac{b}{c}}$

$= \frac{1}{8}x$

$\therefore a^{\frac{1}{2}} = \frac{1}{8}x$

b) $\dfrac{16}{b^3} = \dfrac{16}{\left(\frac{16}{\sqrt{x}}\right)^3} = \dfrac{16}{\left(\frac{16}{x^{1/2}}\right)^3} = \dfrac{16}{\left(\frac{16^3}{x^{1/2 \times 3}}\right)} = \dfrac{16}{\left(\frac{16^3}{x^{3/2}}\right)}$

indicies rule: $(a^b)^c = a^{bc}$

$= \dfrac{16}{\left(\frac{4096}{x^{3/2}}\right)} = \dfrac{16}{1} \div \dfrac{4096}{x^{3/2}} = \dfrac{16}{1} \times \dfrac{x^{3/2}}{4096} = \dfrac{16}{4096}x^{3/2}$

dividing by fractions 'keep' 'change' 'flip'

keep change flip

$\therefore \dfrac{16}{b^3} = \dfrac{1}{256}x^{3/2}$

c) $\left(\dfrac{ab}{2}\right)^{-4/3} = \left(\dfrac{\frac{1}{64}x^2 \times \frac{16}{\sqrt{x}}}{2}\right)^{-4/3} = \left(\dfrac{\left(\frac{16x^2}{64\sqrt{x}}\right)}{2}\right)^{-4/3} = \left(\dfrac{16x^2}{128\sqrt{x}}\right)^{-4/3}$

$= \left(\dfrac{x^2}{8\sqrt{x}}\right)^{-4/3} = \left(\dfrac{x^2}{8x^{1/2}}\right)^{-4/3} = \left(\dfrac{x^{1/2}(x^{3/2})}{x^{1/2}(8)}\right)^{-4/3}$

indicies rule: $a^{b+c} = a^b \times a^c$

indicies rule: $(a^b)^c = a^{bc}$

**Question 2 continued**

$$= \left(\frac{x^{3/2}}{8}\right)^{-4/3} = \left(\frac{x^{3/2 \times 4/3}}{8^{4/3}}\right)^{-1} = \left(\frac{x^2}{16}\right)^{-1} = \frac{16}{x^2}$$

indicies rule: $\left(\frac{a}{b}\right)^{-1} = \left(\frac{b}{a}\right)^{1}$

$$= 16x^{-2}$$

indicies rule: $\frac{a}{x^b} = ax^{-b}$

$$\therefore \left(\frac{ab}{2}\right)^{-4/3} = 16x^{-2}$$

**(Total for Question 2 is 4 marks)**

DO NOT WRITE IN THIS AREA

3. **In this question you must show all stages of your working.**

**Solutions relying on calculator technology are not acceptable.**

(a) Write $\dfrac{8-\sqrt{15}}{2\sqrt{3}+\sqrt{5}}$ in the form $a\sqrt{3}+b\sqrt{5}$ where $a$ and $b$ are integers to be found.

**(3)**

(b) Hence, or otherwise, solve

$$\left(x+5\sqrt{3}\right)\sqrt{5}=40-2x\sqrt{3}$$

giving your answer in simplest form.

**(3)**

a) rationalising surds

$$\frac{8-\sqrt{15}}{2\sqrt{3}+\sqrt{5}} \begin{matrix}\times(2\sqrt{3}-\sqrt{5})\\ \times(2\sqrt{3}-\sqrt{5})\end{matrix} = \frac{(8-\sqrt{15})(2\sqrt{3}-\sqrt{5})}{(2\sqrt{3}+5)(2\sqrt{3}-\sqrt{5})}$$

$$= \frac{16\sqrt{3}-8\sqrt{5}-2\sqrt{3\times15}+\sqrt{15\times5}}{12-2\sqrt{3\times5}+2\sqrt{3\times5}-5} = \frac{16\sqrt{3}-8\sqrt{5}-2\sqrt{45}+\sqrt{75}}{12-5}$$

$$= \frac{16\sqrt{3}-8\sqrt{5}-2\sqrt{9\times5}+\sqrt{25\times3}}{7} = \frac{16\sqrt{3}-8\sqrt{5}-2(3)\sqrt{5}+5\sqrt{3}}{7}$$

$$= \frac{16\sqrt{3}-8\sqrt{5}-6\sqrt{5}+5\sqrt{3}}{7} = \frac{21\sqrt{3}-14\sqrt{5}}{7} = \frac{7(3\sqrt{3}-2\sqrt{5})}{7(1)}$$

$\therefore\ 3\sqrt{3}-2\sqrt{5}$ $\quad a=3 \quad b=-2$

b) Make $x$ the subject :

$$(x+5\sqrt{3})\sqrt{5} = 40-2x\sqrt{3}$$
$$x\sqrt{5}+5\sqrt{3\times5} = 40-2x\sqrt{3}$$
$$x\sqrt{5}+5\sqrt{15} = 40-2x\sqrt{3} \quad +2x\sqrt{3}$$
$$+2x\sqrt{3} \quad x\sqrt{5}+2x\sqrt{3}+5\sqrt{15} = 40$$
$$-5\sqrt{15} \quad x\sqrt{5}+2x\sqrt{3} = 40-5\sqrt{15} \quad -5\sqrt{15}$$

$$\div(\sqrt{5}+2\sqrt{3}) \quad x(\sqrt{5}+2\sqrt{3}) = 40-5\sqrt{15} \quad \div(\sqrt{5}+2\sqrt{3})$$
$$x = \frac{40-5\sqrt{15}}{2\sqrt{3}+\sqrt{5}}$$

**Question 3 continued**

$$x = \frac{40 - 5\sqrt{15}}{2\sqrt{3} + \sqrt{5}} = \frac{5(8 - \sqrt{15})}{2\sqrt{3} + \sqrt{5}}$$

$$= 5\left(\frac{8 - \sqrt{15}}{2\sqrt{3} + \sqrt{5}}\right)$$ from part (a)

$\therefore$ after rationalising: $x = 5(3\sqrt{3} - 2\sqrt{5}) = 15\sqrt{3} - 10\sqrt{5}$

$\therefore$ $x = 15\sqrt{3} - 10\sqrt{5}$

**(Total for Question 3 is 6 marks)**

**4.**

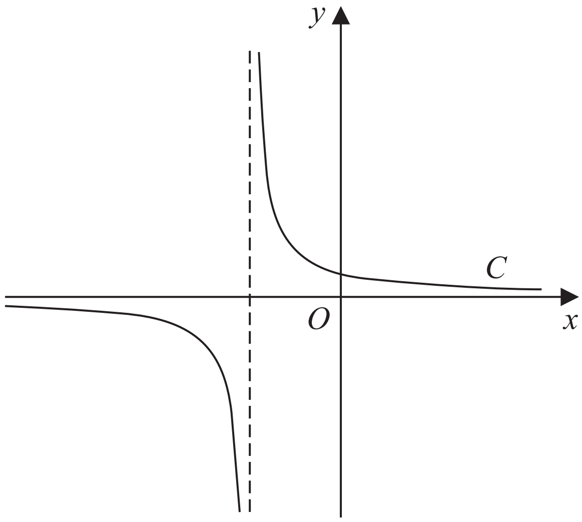



**Figure 1**

Figure 1 shows a sketch of part of the curve $C$ with equation $y = \dfrac{1}{x+2}$

(a) State the equation of the asymptote of $C$ that is parallel to the $y$-axis.

**(1)**

(b) Factorise fully $x^3 + 4x^2 + 4x$

**(2)**

A copy of Figure 1, labelled Diagram 1, is shown on the next page.

(c) On Diagram 1, add a sketch of the curve with equation

$$y = x^3 + 4x^2 + 4x$$

On your sketch, state clearly the coordinates of each point where this curve cuts or meets the coordinate axes.

**(3)**

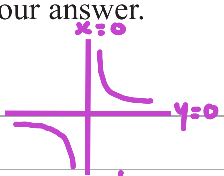
(d) Hence state the number of real solutions of the equation

$$(x+2)(x^3 + 4x^2 + 4x) = 1$$

giving a reason for your answer.

**(1)**

a) in $y = \frac{1}{x}$




asymptotes are $x = 0$ & $y = 0$

when inside f(x) brackets, we do the inverse

if $f(x) = \frac{1}{x}$ $f(x+2) = \frac{1}{x+2}$ which means translation $\binom{-2}{0}$, to the left by 2 units

As +2 is inside brackets, y-coordinates & asymptote is unchanged.

$\therefore x = 0 - 2 = -2$

$\therefore x = -2$

Question 4 continued

b) $x^3+4x^2+4x^1 = x(x^2+4x+4) = x((x+2)(x+2)) = x((x+2)^2)$

factorise quadratic equation

$\therefore x(x+2)^2$

c) working out: $y=x^3+4x^2+4x$ / $y=x(x+2)^2$

- graph is cubic as there is an $x^3$ ∴ shape is ∿ or ∿.
- graph is positive cubic ∵ $x^3$ is positive ∴ ∿ shape
- when $x=0$, intersects y-axis at $y=0(0+2)^2=0$ ∴ $(0,0)$
- $y=x(x+2)^2$ squared so will 'meet' point $x+2=0 \rightarrow x=-2$ ∴ meets at $(-2,0)$

linear so will 'cut' point $x=0$ ∴ cuts at $(0,0)$

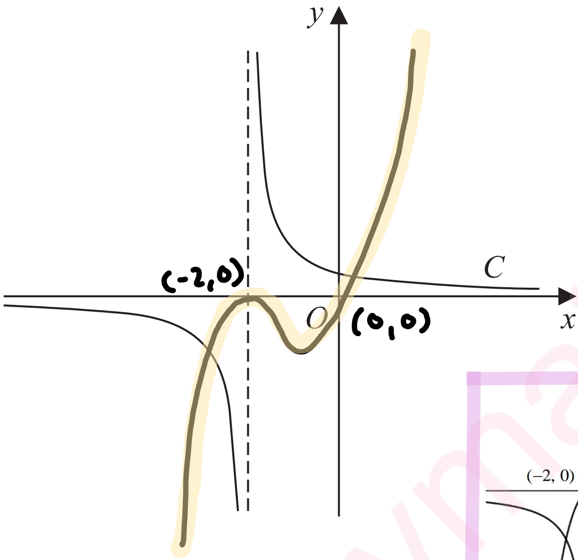
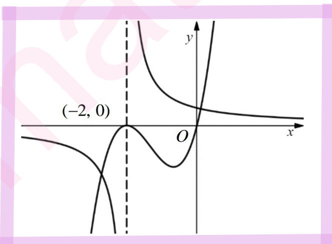
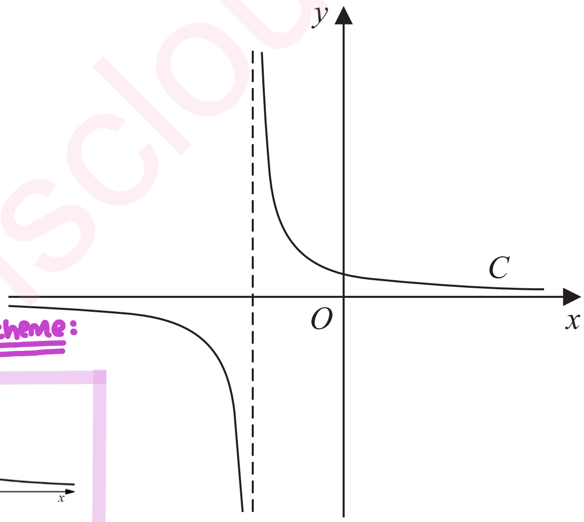



Diagram 1

mark scheme:



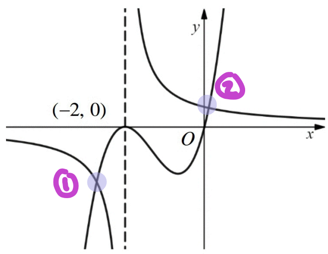




copy of Diagram 1

Only use the copy of Diagram 1 if you need to redraw your answer to part (c).

d) $(x+2)(x^3+4x^2+4x)=1$ $\div(x+2)$

$\div(x+2)$ $x^3+4x^2+4x = \dfrac{1}{x+2}$




2 intersections

∴ 2 real solutions

∴ 2 real solutions ∵ $(x+2)(x^3+4x^2+4x)=1$ is the same as $x^3+4x^2+4x = \dfrac{1}{x+2}$ & the graphs intersect each other twice.

**(Total for Question 4 is 7 marks)**

**5.**

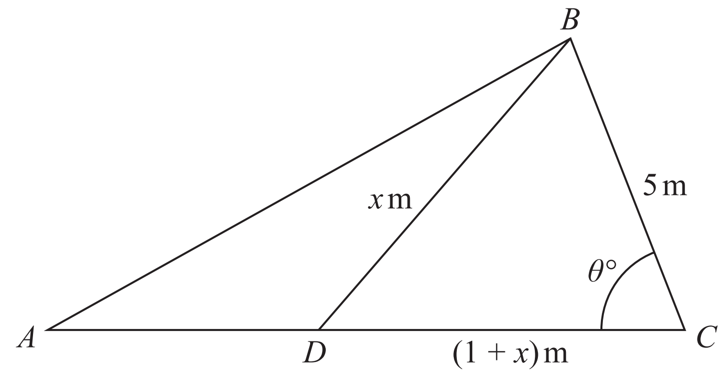



**Figure 2**

Figure 2 shows the plan view of a frame for a flat roof.

The shape of the frame consists of triangle $ABD$ joined to triangle $BCD$.

Given that

- $BD = x$ m
- $CD = (1 + x)$ m
- $BC = 5$ m
- angle $BCD = \theta°$ → unit is degrees

(a) show that $\cos\theta° = \dfrac{13 + x}{5 + 5x}$

**(2)**

Given also that

- $x = 2\sqrt{3}$
- angle $BAC = 30°$
- $ADC$ is a straight line

(b) find the area of triangle $ABC$, giving your answer, in $m^2$, to one decimal place.

**(5)**

a) Cosine rule :

**Pure Mathematics P1**

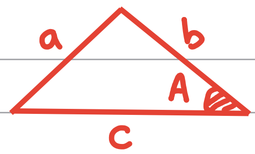
**Mensuration**

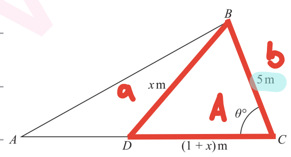
Surface area of sphere $= 4\pi r^2$

Area of curved surface of cone $= \pi r \times$ slant height

**Cosine rule**

$a^2 = b^2 + c^2 - 2bc\cos A$




$x^2 = 5^2 + (1+x)^2 - 2(5)(1+x)\cos\theta$

$x^2 = 25 + (1+x+x+x^2) - 2(5+5x)\cos\theta$

$x^2 = 25 + (1+2x+x^2) - (10+10x)\cos\theta$

$-25$ ↳ $x^2 - 25 = (1+2x+x^2) - (10+10x)\cos\theta$ $-25$

$-(1+2x+x^2)$ ↳ $x^2 - 25 - (1+2x+x^2) = -(10+10x)\cos\theta$ $-(1+2x+x^2)$

$x^2 - 25 - 1 - 2x - x^2 = -(10+10x)\cos\theta$

Question 5 continued

$$-26-2x = -(10+10x)\cos\theta$$

$\div -1$ $\quad$ $\div -1$

$$26+2x = (10+10x)\cos\theta$$

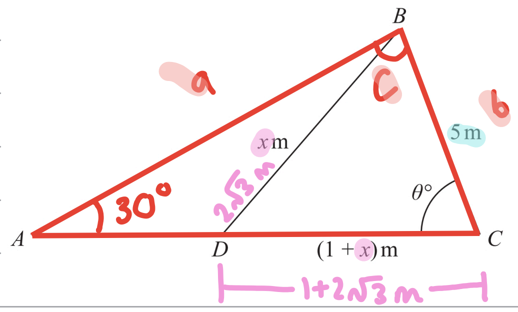
$\div(10+10x)$ $\quad$ $\div(10+10x)$

$$\frac{26+2x}{10+10x} = \cos\theta$$

$$\cos\theta = \frac{2(13+x)}{2(5+5x)} = \frac{13+x}{5+5x}$$

$$\therefore \cos\theta = \frac{13+x}{5+5x}$$

b)




$x = 2\sqrt{3}$ m

Area of a triangle : $A = \frac{1}{2}ab\sin C$

$$A = \frac{1}{2} \times AB \times BC \times \sin\angle ABC$$

① find angle ABC.

Total area in a triangle is 180° $\therefore$ $180 = \angle BAC + \angle BCA + \angle ABC$

$180 = 30 + \angle BCA + \angle ABC$

$$\cos\theta = \cos\angle BCA = \frac{13+x}{5+5x} \quad \text{(from part (a))}$$

find $\cos\angle BCA$ by doing $x = 2\sqrt{3}$

$$\cos\angle BCA = \frac{13+(2\sqrt{3})}{5+5(2\sqrt{3})} = \frac{13+2\sqrt{3}}{5+10\sqrt{3}}$$

$$\theta = \angle BCA = \cos^{-1}\left(\frac{13+2\sqrt{3}}{5+10\sqrt{3}}\right) = 42.47074\ldots \approx 42.47°$$

$$180 = 30 + 42.47 + \angle ABC$$

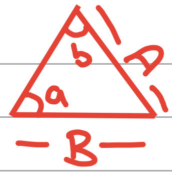
$$180 = 72.47 + \angle ABC$$

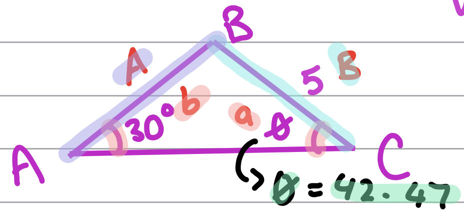
$-72.47$ $\quad$ $-72.47$

$$107.53° = \angle ABC$$

**Question 5 continued**

(2) find side AB using sine rule $\frac{\sin a}{A} = \frac{\sin b}{B}$




$$\frac{\sin 42.47}{AB} = \frac{\sin 30}{5}$$

$$\frac{\sin 42.47}{AB} = \frac{1}{10}$$

×10 ×10

$$\frac{10\sin 42.47}{AB} = 1$$

×AB ×AB

$$10\sin 42.47 = AB$$

$AB = 10\sin 42.47 = 6.752040\ldots \approx 6.75$

(3) $A = \frac{1}{2} \times AB \times BC \times \sin \angle ABC$

$A = \frac{1}{2} \times 6.75 \times 5 \times \sin 107.53 = 16.0913\ldots \approx 16.1$

$\therefore$ Area $= 16.1\ m^2$ (1dp)

**Question 5 continued**

**(Total for Question 5 is 7 marks)**

6. **In this question you must show all stages of your working.**

**Solutions relying on calculator technology are not acceptable.**

The equation

$$4(p - 2x) = \frac{12 + 15p}{x + p} \qquad x \neq -p$$

where $p$ is a constant, has two distinct real roots.

(a) Show that

$$3p^2 - 10p - 8 > 0$$

**(3)**

(b) Hence, using algebra, find the range of possible values of $p$

**(3)**

a) Using discriminant rule $b^2 - 4ac > 0$

① form quadratic equation

$$4(p-2x) = \frac{12+15p}{x+p}$$

$$4p - 8x = \frac{12+15p}{x+p}$$

$\times(x+p)$ $\times(x+p)$

$$(4p-8x)(x+p) = 12+15p$$

$$4px + 4p^2 - 8x^2 - 8px = 12 + 15p$$

$-(12+15p)$ $-(12+15p)$

$$4px + 4p^2 - 8x^2 - 8px - 12 - 15p = 0$$

$$-8x^2 - 4px + (4p^2 - 15p - 12) = 0$$

$\times -1$ $\times -1$

$$8x^2 + 4px + (-4p^2 + 15p + 12) = 0$$

② two real roots means that when the equation of the graph is $ax^2 + bx + c = 0$
discriminant is $b^2 - 4ac > 0$

$$8x^2 + 4px + (-4p^2 + 15p + 12) = 0$$

$$b^2 - 4ac > 0$$

$$(4p)^2 - 4(8)(-4p^2 + 15p + 12) > 0$$

**Question 6 continued**

③ expand & Simplify

$(4p)^2 - 4(8)(-4p^2+15p+12) > 0$

$16p^2 - (32)(-4p^2+15p+12) > 0$

$16p^2 - (-128p^2+480p+384) > 0$

−ve x −ve = +ve

$16p^2 + 128p^2 - 480p - 384 > 0$

$144p^2 - 480p - 384 > 0$

÷48 ÷48

$3p^2 - 10p - 8 > 0$

$\therefore 3p^2 - 10p - 8 > 0$

b) find critical values

factorise $3p^2 - 10p - 8 = 0$

$(3p+2)(p-4) = 0$

Solve : • $3p+2=0$ • $p-4=0$

$3p = -2$ $\therefore p = 4$

$\therefore p = -2/3$

Critical values are $p = -2/3,\ 4$

when P is one more than $-2/3$, $p = 1/3$

$3(1/3)^2 - 10(1/3) - 8 > 0$

$-11 > 0$

this is false $\therefore$ P cannot be greater than $-2/3$

$\therefore p < -2/3$

When P is one less than 4, $p = 3$

$3(3)^2 - 10(3) - 8 > 0$

$-11 > 0$

this is false $\therefore$ P cannot be less than 4

$\therefore p > 4$

$\therefore p > 4$ & $p < -2/3$

**(Total for Question 6 is 6 marks)**

**7.** The curve $C$ has equation $y = \mathrm{f}(x)$ where $x > 0$

Given that

- $\mathrm{f}'(x) = \dfrac{4x^2 + 10 - 7x^{\frac{1}{2}}}{4x^{\frac{1}{2}}}$
- the point $P(4, -1)$ lies on $C$

(a) (i) find the value of the gradient of $C$ at $P$

(ii) Hence find the equation of the normal to $C$ at $P$, giving your answer in the form $ax + by + c = 0$ where $a$, $b$ and $c$ are integers to be found.

**(4)**

(b) Find $\mathrm{f}(x)$.

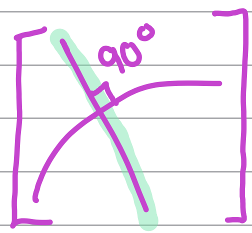
**(6)**

a)i) $f'(x)$ is gradient function, so by substituting $x$-coordinate of $P$ you can find gradient of $C$ at $P$.

$$f'(4) = \frac{4(4)^2 + 10 - 7(4)^{1/2}}{4(4)^{1/2}} = \frac{4(16) + 10 - 7(2)}{4(2)} = \frac{60}{8} = \frac{15}{2}$$

$\therefore$ gradient of $C$ at $P$ is $\frac{15}{2}$

ii) normal is Perpendicular to Curve




$\therefore$ we find gradient of normal ($m_n$) using Perpendicular gradient rule $M_{normal} \times M_{curve} = -1$

① find gradient of normal at $C$.

$$m_n \times m_C = -1$$

$$\div \tfrac{15}{2} \curvearrowleft \quad m_n \times 15/2 = -1 \quad \curvearrowright \div \tfrac{15}{2}$$

$$m_n = -2/15$$

② find equation of normal using line passing through $(a, b)$ & gradient $M$

equation: $(y - b) = M(x - a)$

$a = 4$

$b = -1$

$M = -2/15$

$$(y - (-1)) = -\frac{2}{15}(x - 4)$$





P 7 4 3 1 6 A 0 1 6 3 2

Question 7 continued

(3) Write in form $ax + by + c = 0$

$-ve \times -ve = +ve$

$$y + 1 = -\tfrac{2}{15}(x - 4)$$

$\times 15$ $\qquad$ $\times 15$

$$15y + 15 = -2(x - 4)$$

$+2x$ $\qquad$ $+2x$

$$15y + 15 = -2x + 8$$

$$15y + 2x + 15 = 8$$

$-8$ $\qquad$ $-8$

$$15y + 2x + 7 = 0$$

$$\therefore\ 2x + 15y + 7 = 0$$

b) $f(x) \underset{\text{differentiate}}{\overset{\text{integrate}}{\rightleftarrows}} f'(x)$

(1) write $f'(x)$ in easier form for integration

$\because x^0 = 1$

$$f'(x) = \frac{4x^2 + 10 - 7x^{1/2}}{4x^{1/2}} = \frac{4x^2}{4x^{1/2}} + \frac{10}{4x^{1/2}} - \frac{7x^{1/2}}{4x^{1/2}} = 1\,\frac{x^2}{x^{1/2}} + \frac{5}{2}\,\frac{x^0}{x^{1/2}} - \frac{7}{4}\,\frac{x^{1/2}}{x^{1/2}}$$

$$= \frac{x^2}{x^{1/2}} + \frac{5}{2}\,\frac{x^0}{x^{1/2}} - \frac{7}{4} = x^{2-1/2} + \frac{5}{2}x^{0-1/2} - \frac{7}{4} = x^{3/2} + \frac{5}{2}x^{-1/2} - \frac{7}{4}$$

* indicies rule : $\dfrac{a^b}{a^c} = a^{b-c}$

(2) Integrate

$$f(x) = \int f'(x)\,dx = \int x^{3/2} + \frac{5}{2}x^{-1/2} - \frac{7}{4}x^0\,dx = \left[\left(\frac{1}{\frac{3}{2}+1}x^{3/2+1}\right) + \left(\frac{\frac{5}{2}}{-\frac{1}{2}+1}x^{-1/2+1}\right) + \left(\frac{-7/4}{0+1}x^{0+1}\right)\right]$$

$$= \frac{2}{5}x^{5/2} + 5x^{1/2} - \frac{7}{4}x^1 + c$$

(3) find $+c$ by substituting $P(4, -1)$, so that $f(4) = -1$

$$f(4) = \frac{2}{5}(4)^{5/2} + 5(4)^{1/2} - \frac{7}{4}(4) + c = -1$$

$$\frac{79}{5} + c = -1$$

$-79/5$ $\qquad$ $-79/5$

$$c = -\frac{84}{5}$$

$$\therefore\ f(x) = \frac{2}{5}x^{5/2} + 5x^{1/2} - \frac{7}{4}x - \frac{84}{5}$$

**Question 7 continued**

**Question 7 continued**

**(Total for Question 7 is 10 marks)**

**8.** **In this question you must show all stages of your working.**

**Solutions relying on calculator technology are not acceptable.**

The curve $C_1$ has equation

$$xy = \frac{15}{2} - 5x \qquad x \neq 0$$

The curve $C_2$ has equation

$$y = x^3 - \frac{7}{2}x - 5$$

(a) Show that $C_1$ and $C_2$ meet when

$$2x^4 - 7x^2 - 15 = 0$$

**(2)**

Given that $C_1$ and $C_2$ meet at points $P$ and $Q$

(b) find, using algebra, the exact distance $PQ$

**(5)**

a) ① rearrange $C_1$ so that $y$ is the subject

$$xy = \frac{15}{2} - 5x$$

$\div x$ $\quad y = \frac{15}{2x} - 5$ $\quad \div x$

② equate $C_1$ & $C_2$ to find equation for point(s) where $C_1$ & $C_2$ meet.

$$C_1 = C_2$$

$$\frac{15}{2x} - 5 = x^3 - \frac{7}{2}x - 5$$

$\times 2x$ $\quad 15 - 10x = 2x^4 - 7x^2 - 10x$ $\quad \times 2x$

$+10x$ $\quad 15 = 2x^4 - 7x^2$ $\quad +10x$

$-15$ $\quad 0 = 2x^4 - 7x^2 - 15$ $\quad -15$

$$\therefore 2x^4 - 7x^2 - 15 = 0$$

b) ① Solve $2x^4 - 7x^2 - 15 = 0$ to find $x$-coordinates of $P$ & $Q$

let $x^2 = a$ $\qquad x^4 = (x^2)^2 = a^2$

$$\therefore 2a^2 - 7a - 15 = 0$$

factorise: $(2a + 3)(a - 5) = 0$

Solve: • $2a + 3 = 0$ $\qquad$ • $a - 5 = 0$

$2a = -3$ $\qquad \therefore a = 5$

$\therefore a = -\frac{3}{2}$

**Question 8 continued**

let $a = x^2$ so when $a = -3/2$

let $x^2 \neq -3/2$

UNDEFINED $\because$ Cannot have negative square number

when $a = 5$

let $x^2 = 5$

$x = \pm\sqrt{5}$

(2) Substitute $x = \pm\sqrt{5}$ into equation $C_1$ or $C_2$ to find y-coordinates

when $x = +\sqrt{5}$ $C_2$: $y = \frac{15}{2(\sqrt{5})} - 5 = \frac{3\sqrt{5}}{2} - 5 = \frac{3}{2}\sqrt{5} - 5$

when $x = -\sqrt{5}$ $y = \frac{15}{2(-\sqrt{5})} - 5 = -\frac{3\sqrt{5}}{2} - 5 = -\frac{3}{2}\sqrt{5} - 5$

$\therefore$ $C_1$ & $C_2$ meet at $(\sqrt{5}, \frac{3}{2}\sqrt{5} - 5)$ & $(-\sqrt{5}, -\frac{3}{2}\sqrt{5} - 5)$

(3) distance between 2 points: $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$(\sqrt{5}, \frac{3}{2}\sqrt{5} - 5)$ & $(-\sqrt{5}, -\frac{3}{2}\sqrt{5} - 5)$

distance $= |PQ| = \sqrt{(\sqrt{5} - (-\sqrt{5}))^2 + ((\frac{3}{2}\sqrt{5} - 5) - (-\frac{3}{2}\sqrt{5} - 5))^2}$

$= \sqrt{(2\sqrt{5})^2 + (3\sqrt{5})^2} = \sqrt{65}$

$\therefore$ distance $PQ = \sqrt{65}$

**Question 8 continued**

**Question 8 continued**

**(Total for Question 8 is 7 marks)**

**9.**

Diagram **NOT** accurately drawn

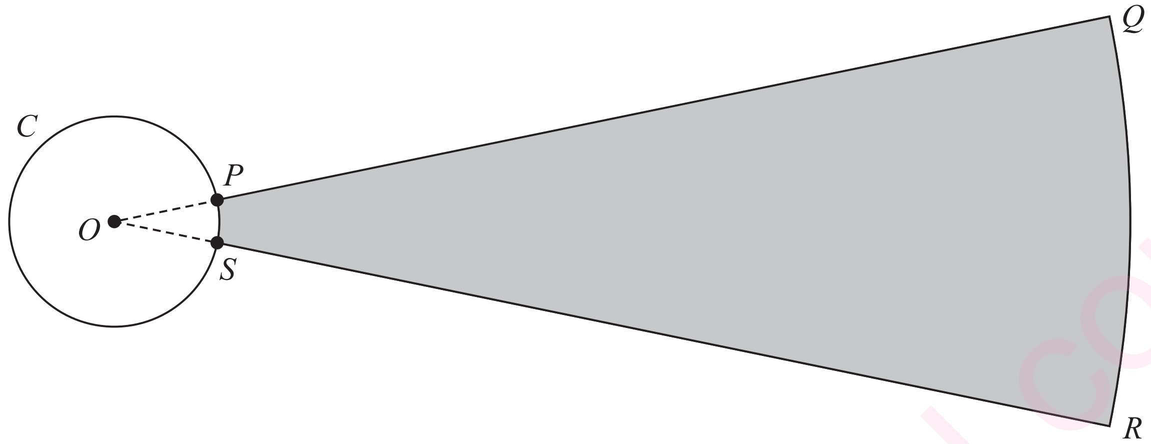



**Figure 3**

Figure 3 shows the plan view of the area being used for a ball-throwing competition.

Competitors must stand within the circle $C$ and throw a ball as far as possible into the target area, $PQRS$, shown shaded in Figure 3.

Given that

- circle $C$ has centre $O$
- $P$ and $S$ are points on $C$
- $OPQRSO$ is a sector of a circle with centre $O$
- the length of arc $PS$ is 0.72 m
- the size of angle $POS$ is 0.6 radians

(a) show that $OP = 1.2$ m

radius of circle C

**(1)**

Given also that

- the target area, $PQRS$, is $90\,\text{m}^2$
- length $PQ = x$ metres

(b) show that

$$5x^2 + 12x - 1500 = 0$$

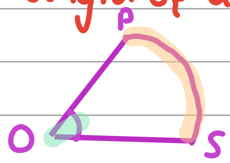
**(3)**

(c) Hence calculate the total perimeter of the target area, $PQRS$, giving your answer to the nearest metre.

**(3)**

Question 9 continued

a) Length of arc : $S = r\theta$



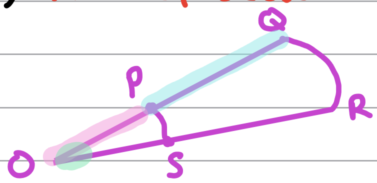

Arc PS : $PS = OP \times \angle POS$

$\div 0.6$ $\quad 0.72 = OP \times 0.6 \quad$ $\div 0.6$

$1.2 = OP$

$\therefore OP = 1.2m$

b) Area of Sector : $A = \frac{1}{2} r^2 \theta$




Area PQRS = Area OPQRS - Area OPS = 90

① Area OPQRS : $A = \frac{1}{2} \times OPQ^2 \times \angle POS = \frac{1}{2} \times (1.2 + x)^2 \times 0.6$

$= 0.3(1.44 + 2.4x + x^2) = 0.432 + 0.72x + 0.3x^2$

② Area OPS : $A = \frac{1}{2} \times OP^2 \times \angle POS = \frac{1}{2} \times (1.2)^2 \times 0.6 = 0.432$

$A_{PQRS} = (0.3x^2 + 0.72x + 0.432) - 0.432 = 90$

$0.3x^2 + 0.72x = 90$

$-90 \quad 0.3x^2 + 0.72x - 90 = 0 \quad -90$

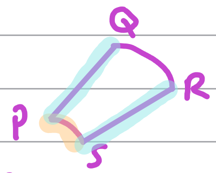
$\div 0.3 \quad x^2 + 2.4x - 300 = 0 \quad \div 0.3$

$\times 5 \quad 5x^2 + 12x - 1500 = 0 \quad \times 5$

$\therefore 5x^2 + 12x - 1500 = 0$

c) Perimeter of PQRS = PQ + QR + RS + PS

$= x + QR + x + 0.72$




① find value of $x$ by solving $5x^2 + 12x - 1500 = 0$ (from part (a))

Solve using quadratic formula $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$a = 5$

$b = -12$

$c = -1500$

$\frac{-(12) \pm \sqrt{(-12)^2 - 4(5)(-1500)}}{2(5)} = \frac{-12 \pm 8\sqrt{471}}{2(5)}$

$= \frac{-6 \pm 4\sqrt{471}}{5}$

$x_1 = \frac{-6 + 4\sqrt{471}}{5} = 16.162027... \approx 16.16$

$x_2 \neq \frac{-6 - 4\sqrt{471}}{5} = -18.5620...$

negative number cannot be value of a length

**Question 9 continued**

$\therefore x = 16.16m$

② find length of arc QR.

$S = r\theta$

$S = (1.2 + x) \times 0.6 = (1.2 + 16.16) \times 0.6 = 10.416$

③ Perimeter = PQ + QR + RS + PS

$16.16 + 10.416 + 16.16 + 0.72$

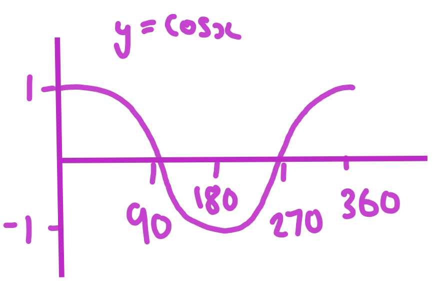
$= 43.456$

$\therefore$ Perimeter = 43m (to the nearest metre)

**Question 9 continued**

**(Total for Question 9 is 7 marks)**

**10.**



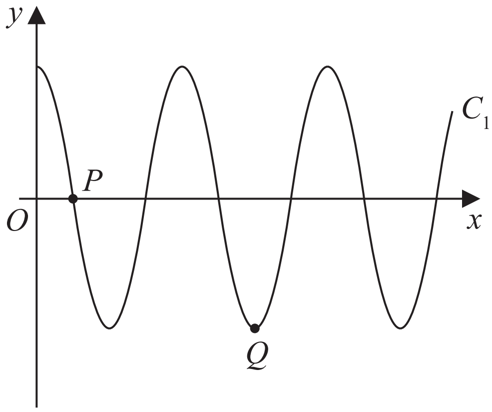




**Figure 4**

Figure 4 shows a sketch of part of the curve $C_1$ with equation

$$y = 3\cos\left(\frac{x}{n}\right)^\circ \qquad x \geqslant 0$$

where $n$ is a constant.

The curve $C_1$ cuts the positive $x$-axis for the first time at point $P(270, 0)$, as shown in Figure 4.

(a) (i) State the value of $n$

(ii) State the period of $C_1$

**(2)**

The point $Q$, shown in Figure 4, is a minimum point of $C_1$

(b) State the coordinates of $Q$.

**(2)**

The curve $C_2$ has equation $y = 2\sin x^\circ + k$, where $k$ is a constant.

The point $R\left(a, \frac{12}{5}\right)$ and the point $S\left(-a, -\frac{3}{5}\right)$, both lie on $C_2$

Given that $a$ is a constant less than 90

(c) find the value of $k$.

**(2)**

a)i) in $y = \cos x$, curve cuts positive $x$-axis at $(90, 0)$.
if $f(x) = \cos x$, $3f\left(\frac{x}{n}\right) = 3\cos\left(\frac{x}{n}\right)$ translation is vertical stretch by 3 units & horizontal stretch by $n$ units

$\therefore$ for point $(90, 0) \Rightarrow (90 \times n, 0 \times 3) = (90n, 0)$
$(90n, 0) = (270, 0)$

$\hookrightarrow 90n = 270$, $n = 3$ $\hookleftarrow$ $\qquad \therefore n = 3$

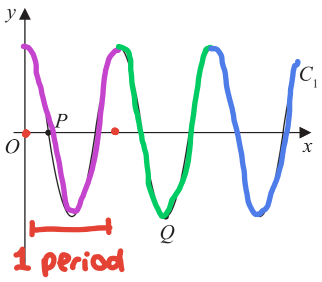




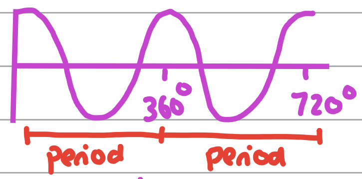
P 7 4 3 1 6 A 0 2 8 3 2

**Question 10 continued**

ii) Period is every time graph shape repeats.




in graph of $y = \cos x$, period is $360°$.

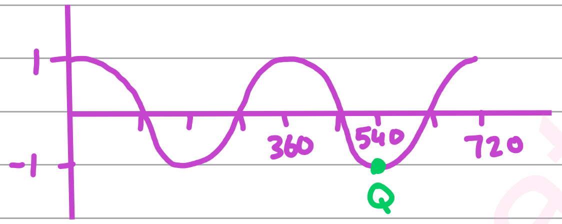



$\therefore$ for $y = 3\cos\left(\frac{x}{n}\right)$ : $360° \times n$

$360 \times 3 = 1080°$

$\therefore$ Period is $1080°$

b) Point Q on graph of $y = \cos x$ is $2^{nd}$ minimum on positive $x$-axis:




$Q\,(540°, -1)$

for $y = 3\cos\left(\frac{x}{n}\right)$ $\quad Q\,(540 \times 3,\ -1 \times 3)$

$\therefore Q\,(1620°, -3)$

c) form 2 equations:

for $\left(a, \frac{12}{5}\right)$ $\quad 2\sin a + K = \frac{12}{5}$ $\qquad -(\sin x) = \sin(-x)$

for $\left(-a, -\frac{3}{5}\right)$ $\quad 2\sin -a + K = -\frac{3}{5}$

$$
\begin{array}{r}
2\sin a + K = \frac{12}{5} \\
-\,(2\sin a) + K = -\frac{3}{5} \\
\hline
0 \quad + 2K = \frac{9}{5}
\end{array}
\quad +
$$

$\div 2$ $\quad 2K = 9/5$ $\quad \div 2$

$K = 9/10$

$\therefore K = \frac{9}{10}$

**(Total for Question 10 is 6 marks)**

**11.**

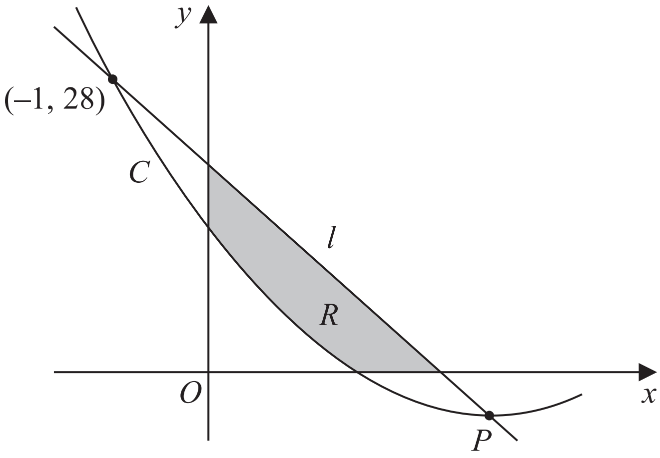



**Figure 5**

Figure 5 shows part of the curve $C$ with equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = 2x^2 - 12x + 14$$

(a) Write $2x^2 - 12x + 14$ in the form

$$a(x + b)^2 + c$$

where $a$, $b$ and $c$ are constants to be found.

**(3)**

Given that $C$ has a minimum at the point $P$

(b) state the coordinates of $P$

**(1)**

The line $l$ intersects $C$ at (–1, 28) and at $P$ as shown in Figure 5.

(c) Find the equation of $l$ giving your answer in the form $y = mx + c$ where $m$ and $c$ are constants to be found.

**(3)**

The finite region $R$, shown shaded in Figure 5, is bounded by the $x$-axis, $l$, the $y$-axis, and $C$.

(d) Use inequalities to define the region $R$.

**(3)**

a) Completing the square: if $y = x^2 + bx + C$

$$y = \left(x + \frac{b}{2}\right)^2 + C - \left(\frac{b}{2}\right)^2$$

① write in form $a(x^2 + bx + c)$

$$2x^2 - 12x + 14 = 2(x^2 - 6x + 7)$$

**Question 11 continued**

② Completing the square

$b = -6$

$C = 7$

$y = 2\left(\left(x + \frac{(-6)}{2}\right)^2 + 7 - \left(\frac{-6}{2}\right)^2\right)$

$y = 2\left((x-3)^2 + 7 - 9\right)$

$y = 2\left((x-3)^2 - 2\right)$

$\therefore y = 2(x-3)^2 - 4$ $\quad\therefore a = 2 \quad b = -3 \quad c = -4$

b) to find coordinates of minimum point:

$y = \underbrace{\left(x + \frac{b}{2}\right)^2}_{\text{inverse is } x\text{-coordinate}} + \underbrace{c - \left(\frac{b}{2}\right)^2}_{y\text{-coordinate}}$

Explanation: when $y = c - (b/2)^2$ $\quad c - (b/2)^2 = (x + b/2)^2 + c - (b/2)^2$

$0 = (x + b/2)^2$

$\therefore x = -b/2$

when $x = -b/2$ $\quad y = (-b/2 + b/2)^2 + c - (b/2)^2$

$\therefore y = c - (b/2)^2$

$y = 2\underbrace{(x-3)^2}_{\text{inverse is } x\text{-coordinate}} \underbrace{-4}_{y\text{-coordinate}}$

$+3 \qquad -4$

$\therefore P(3, -4)$

c) ① find gradient of $l$. gradient formula $m = \frac{y_1 - y_2}{x_1 - x_2}$

$P(3, -4)$ & $(-1, 28)$

$m = \frac{-4 - 28}{3 - (-1)} = \frac{-32}{4} = -8$

② find equation of $l$ using line passing through $(a, b)$ & gradient $m$

equation: $(y - b) = m(x - a)$

$a = 3$

$b = -4$

$m = -8$

$(y - (-4)) = -8(x - 3)$

**Question 11 continued**

③ Write in the form $y = mx + c$

$$y + 4 = -8(x - 3)$$

$$y + 4 = -8x + 24$$

$-4$ $\quad y = -8x + 20 \quad$ $-4$

$\therefore \; l: \; y = -8x + 20$

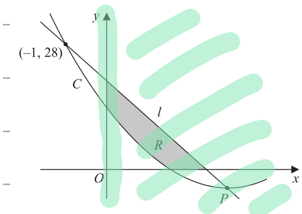
d) 4 inequalities

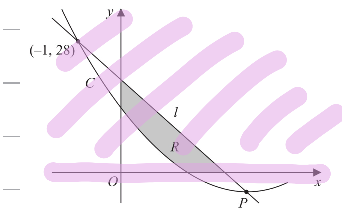
① & ② $\quad y = 0 \;$ & $\; x = 0$

$y \geqslant 0 \;$ & $\; x \geqslant 0$

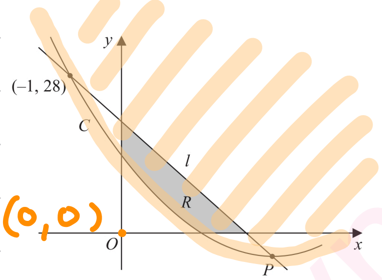
★ $\geqslant$ & not $>$ ∵ lines are Solid & not dashed

Solid dashed




③ $C: \; y = 2x^2 - 12x + 14$




To find inequality, select a point OUTSIDE Valid region and make inequality FALSE.

Point chosen is $(0, 0)$

$0 = 2(0)^2 - 12(0) + 14$

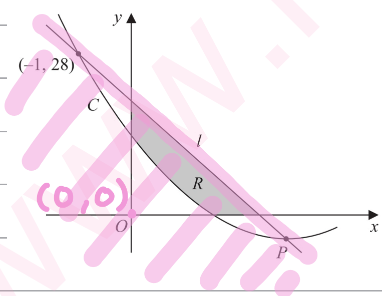
$0 = 14$

Make inequality FALSE: $0 \geqslant 14$

$\therefore \; y \geqslant 2x^2 - 12x + 14$ ★

from part (c)

④ $l: \; y = -8x + 20$




To find inequality, select a point INSIDE Valid region and make inequality TRUE.

Point chosen is $(0, 0)$

$0 = -8(0) + 20$

$0 = 20$

Make inequality TRUE: $0 \leqslant 20$

$\therefore \; y \leqslant -8x + 20$ ★

∴ region R is defined by inequalities:

$y \leqslant -8x + 20 \;, \; y \geqslant 2x^2 - 12x + 14 \;, \; y \geqslant 0 \;, \; x \geqslant 0$

**(Total for Question 11 is 10 marks)**

**TOTAL FOR PAPER IS 75 MARKS**